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Evolutionary dynamics of higher-order interactions in social networks

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We live and cooperate in networks. However, links in networks only allow for pairwise interactions, thus making the framework suitable for dyadic games, but not for games that are played in larger groups. Here, we study the evolutionary dynamics of a public goods game in social systems with higher-order interactions. First, we show that the game on uniform hypergraphs corresponds to the replicator dynamics in the well-mixed limit, providing a formal theoretical foundation to study cooperation in networked groups. Second, we unveil how the presence of hubs and the coexistence of interactions in groups of different sizes affects the evolution of cooperation. Finally, we apply the proposed framework to extract the actual dependence of the synergy factor on the size of a group from real-world collaboration data in science and technology. Our work provides a way to implement informed actions to boost cooperation in social groups.

Cooperation among unrelated individuals distinguishes humans markedly from other mammals, and it is one of the central pillars of our evolutionary success[1]. Past research has emphasized that the structure of social interactions is crucial for the evolution of cooperation, but thus far predominantly in the realm of networks where links connect pairs of players[2,3]. However, since cooperation often unfolds in groups, the need for a paradigm shift in the way we model social interactions is evident and indeed urgent. Regardless of the model that we use to describe human interactions, cooperation remains at odds with the fundamental principles of Darwinian evolution, and it is fascinating that we have succeeded in collectively holding off self-interest over most of the past two million years, ever since the *Homo* genus first emerged[4].

Given this puzzle, the search for reasons and mechanisms that may allow cooperation to evolve and proliferate is an evergreen and vibrant subject across the social and natural sciences[5–11]. Evolutionary game theory is long established as the theory of choice for addressing the puzzle mathematically[12–14], wherein social dilemmas constitute a particularly important class of games. Namely, social dilemmas capture the essence of the problem since defection is the individually optimal strategy, whilst cooperation is the optimal strategy for the highest social welfare[15]. An important mechanism for cooperation in social dilemmas is network reciprocity[16], which stands for the fact that a limited interaction range, as dictated by lattices or other types of network, facilitates the formation of compact clusters of cooperators that are in this way protected against invading defectors. This basic mechanism could also be seen if the degree distribution of the interaction network is strongly heterogeneous[17–19], if there is set or community structure[20,21], or if the evolution unfolds on two or more network layers that mutually support cooperative clusters[22–29].

Despite the wealth of important insights concerning the evolution of cooperation on networks and fundamental discoveries[30–32], accounting for cooperation in groups remains an important unsolved problem, such as in the public goods game (PGG)[33,34]. The simplest remedy is to consider members of a group to be all the players that are pairwise-connected to a central player[35,36]. However, since the other players are further connected in a pairwise manner, one would also need to consider all the groups in which the central player is a member but is not central. Evidently, classical networks do not provide a unique procedure for defining a group. Moreover, members of the same group are commonly not all directly connected with one another, which prevents strategy changes among them, either in terms of imitation, replication or exploration. These facts posit a lack of common theoretical foundation for studying the evolution of cooperation in networked groups. Without knowing who is connected to whom in a group, it is also impossible to implement fundamental mechanisms that promote cooperation, such as reciprocity[37,38], image scoring[39–41] and reputation[42–44].

As a solution, we introduce and study higher-order interactions in evolutionary games that are played in groups. The distinctive feature of higher-order interactions is that, unlike in classical networks[45], a link can connect more than just two individuals[46]. Thus, higher-order networks naturally account for structured group interactions[47], wherein a group is simply made up of all players that are connected by a so-called hyperlink, which is the higher-order analogue of the link. As a paradigmatic example, we consider a standard PGG on the higher-order analogue of a network, referred to as a hypergraph (Fig. 1). We first show that it corresponds exactly to the replicator dynamics in the well-mixed limit as long as no hyperdegree–hyperdegree correlations exist. As such, it thus provides a formal theoretical foundation to study cooperation in networked groups—effectively a null model—that is amenable to further

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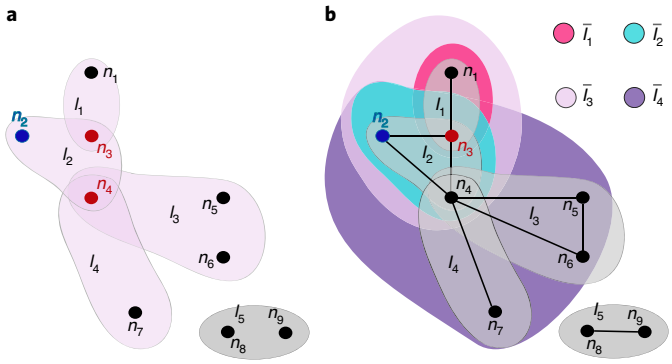

Fig. 1 | Higher-order versus pairwise interactions in a PGG. Comparison of the proposed HI with a standard GI of the game based on pairwise interactions only. **a**, In the HI implementation, a node, n_2, and one of its hyperlinks, l_2, are randomly selected. All the nodes in l_2, namely node n_2, and the two nodes highlighted in red, n_3 and n_4, play all the games they are involved in, corresponding, in this example, to PGG defined for the subset of nodes of the hyperlinks l_1, l_2, l_3 and l_4. Then, the strategy of n_2 is updated by comparing its payoff with that of the node with the highest accumulated payoff of the hyperlink l_2. This is not equivalent to playing the PGG in the graph generated by projecting the interactions of the hypergraph, which is shown in **b**. **b**, In the standard GI implementation, a neighbour of n_2, let us say n_3—highlighted in red—is randomly selected. The two nodes n_2 and n_3 then play all the games of the groups they are part of, that is, of the groups made up by the subsets of nodes $\{n_1, n_3\}$, $\{n_2, n_3, n_4\}$, $\{n_1, n_2, n_3, n_4\}$ and $\{n_2, n_3, n_4, n_5, n_6, n_7\}$. These subsets, coloured as indicated in the figure, could be represented by a different set of hyperlinks $\bar{l}_1$, $\bar{l}_2$, $\bar{l}_3$ and $\bar{l}_4$, respectively, which are different from the set of hyperlinks of the original hypergraph. Finally, the strategy of n_2 is updated by comparing its accumulated payoff to that of node n_3.

upgrades. Next, we consider the PGG on hypergraphs with heterogeneity either in their node hyperdegrees (number of hyperlinks a node is involved into) or in the order of their hyperlinks (number of nodes that form each hyperlink), which allow us to describe the dynamics induced by the presence of highly connected players and to consider scenarios in which the synergy factor depends on the group size in a systematic and consistent way. We show, for example, how synergy factors that are given by different powers of the group size lead to a critical scaling in the transition from defection to cooperation. Lastly, we also demonstrate how the proposed higher-order interaction framework can be used to determine the synergy factor as a function of the group size from empirical data on cooperation and collaborations. Under the assumption that the structure of the hypergraph is the outcome of an optimization process of the game it hosts, we extract the game parameters from datasets describing collaborations in science and technology, showing that higher-order interactions induce diverse benefits and costs in different social domains.

The PGG constitutes the fundamental example of a social dilemma when multiple individuals interact simultaneously. It presents a situation where the gain or loss of an initial investment is shared symmetrically between the members of a group, even if the investment itself can be asymmetric. In other words, there is no correlation between the individual effort and the distribution of the reward, meaning that some players receive more than what they give or deserve, while some others receive less. Metaphorically, one would say that the game has no memory, in the sense that the payoff is assigned blindly to all the players as if the system had lost the information about the original contribution of each player. More formally, the PGG describes a setting where N players are requested to contribute to a common pool with a token of value c (ref. [10]).

Cooperators contribute, and defectors do not. The collected amount is then multiplied by the so-called synergy factor R, and the benefit is shared amongst all the members of the group. The payoff for the defectors and cooperators playing in a group of g members is given by $\pi_D = Rcw_C/g$ and $\pi_C = Rcw_C/g - c$ respectively, with w_C representing the number of cooperators in the group. Typically, c has a fixed value of 1, so that the behaviour of the system is determined by the synergy factor R, or the reduced synergy factor $r = R/g$. It is common to represent the state of the system by the fraction of players adopting each strategy, x_C for the cooperators and x_D for the defectors.

The evolutionary dynamics determine how the strategies of the players evolve with each iteration of the PGG, that is, how the fractions x_C and x_D change with time. Here, we implement the so-called fixed cost per game approach, where cooperators contribute with an entire token to each game they play. Individual updates constitute micro-steps of the dynamics, whereas a (global) time step corresponds to N individual steps, so that all the players in the system have the chance to play the game and update their strategies. Players interact among themselves following the links of the network they are embedded in. As mentioned before, the standard network implementation[35], henceforth referred to as graph implementation (GI), is not able to account for the most general type of interaction in groups. One of the first proposals to overcome the limitation of a GI is evolutionary set theory[20], which considers a structure of interaction in which the players are organized as the elements of a set. Yet, the game itself is pairwise, and thus different from the type of approach proposed here. However, it is worth pointing out that the set theory description is equivalent to the hypergraph formalism, and therefore, one should expect the same results when studying the same game on both structures. In this work we have opted for hypergraphs because, as a higher-order generalization of graphs, they inherit the whole family of graph tools with which evolutionary game theory scholars are more familiar with. A few years later, it was proposed to address higher-order interactions by bipartite graphs, having a set of nodes for the players and a second set for the groups[48–50]. The authors adapted the PGG to the bipartite graph, in what we call the bipartite implementation (BI). In such a case the game is indeed polyadic, but the update process is still dyadic, and the constraints associated with the formalism do not make it suitable for an analytical treatment. Here, we generalize the BI to a fully higher-order implementation and provide the theoretical foundation to study higher-order cooperative games in uniform and heterogeneous hypergraphs. Finally, we mention that in a more recent work[51], the authors have considered games played by agents belonging to subpopulations and whose interactions occur across and within the population, providing a useful methodology for situations in which one can get rid of the fine details of the individual connections.

Results

Game implementation. In order to account for higher-order interactions, we use hypergraphs[46]. A hypergraph, $H(\mathcal{N}, \mathcal{L})$, is a mathematical object that consists of a set of N nodes $\mathcal{N} = \{n_1 = 1, .., n_N = N\}$ and a set of L hyperlinks $\mathcal{L} = \{l_1, ..., l_L\}$. Each hyperlink is a subset of two or more elements of $\mathcal{N}$ and represents a group interaction. For instance, in Fig. 1a, the hyperlink l_1 contains nodes n_1 and n_3, whereas the hyperlink l_3 is the subset made up by nodes n_4, n_5 and n_6. Furthermore, the cardinality of a subset, known as the order of the hyperlink, is the number g of nodes in the group. In the previous example, l_1 has order 2 and l_3 has order 3. In a hypergraph, the hyperdegree, k_i, of a node i represents the number of hyperlinks into which the node is involved, thus, the number of groups of a specific order g that contain i can be denoted by k_i^g. Hence, the hyperdegree of i is given as $k_i = \sum_{g=g^-}^{g^+} k_i^g$, where g^- and g^+ account for the minimal and maximal orders in $\mathcal{L}$. For example, in Fig. 1a,

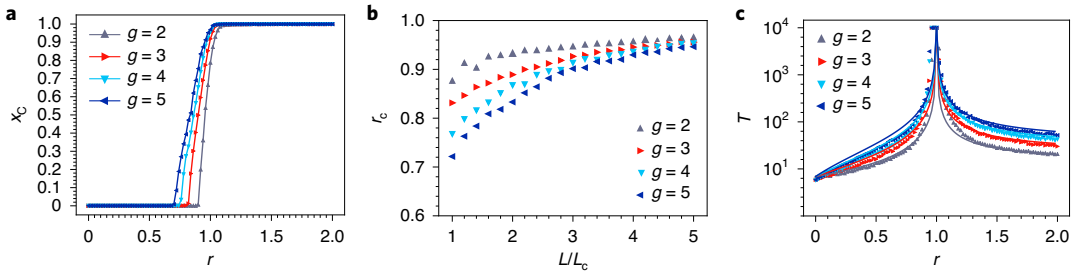

Fig. 2 | A PGG with higher-order interactions in URHs. Numerical simulation of the HI of the game on URHs of $N=1{,}000$ players and different orders g. **a**, Fraction of cooperators, x_c, as a function of the synergy factor, r, for hypergraphs with hyperdegree $\langle k\rangle = k_c$, or total number of hyperlinks $L = L_c$, where k_c and L_c stand for the critical hyperdegree or number of hyperlinks guaranteeing a connected hypergraph. **b**, Critical value of the synergy factor, r_c, as a function of the ratio between the number of hyperlinks L and the critical value L_c in hypergraphs of different density. **c**, Relaxation times as a function of the synergy factor, r, for hypergraphs with hyperdegree $\langle k\rangle = 5k_c$. In all plots, triangles correspond to numerical simulations, while the solid lines are the results of our theoretical predictions.

$k_4 = 3 = k_4^2 + k_4^3$, with $k_4^2 = 1$ (the hyperlink l_4) and $k_4^3 = 2$ (the hyperlinks l_2 and l_3). As $\langle k\rangle$ we indicate the average hyperdegree of node i, where the averages are evaluated over all the nodes in the system, that is $\langle k\rangle = \frac{1}{N}\sum_{i\in\mathcal{N}} k_i$.

Although hypergraphs are not the only possible representation of group interactions, they allow exploitation of the analogy between the links representing pairwise interactions in contact networks and hyperlinks, which are based on higher-order, group interactions. As we will show next, the differences between these two approaches lead to fundamentally distinct outcomes of the PGG evolutionary dynamics. To see how the evolutionary dynamics evolve in hypergraphs, let us consider the first step of a standard graph implementation of the PGG. When a node n_i and one of its neighbours n_j are selected on a graph, it is equivalent to say that a node and one of its links are selected. Such a procedure can be easily generalized to group interactions of more than $g = 2$ individuals, see Fig. 1b. Note that if we choose more neighbours of n_i to generate higher-order interactions, such an extension would still be based on dyadic ones. Instead, we propose a hypergraph implementation (HI) of the game that consists of selecting one of the hyperlinks of n_i. That is, in the HI setup, we select at random with uniform probability a node n_i in the hypergraph and one of its hyperlinks, l_i. Then, all the members of the hyperlink l_i play a game for each of the hyperlinks they are part of, as illustrated in Fig. 1. Finally, as it is customary, the nodes accumulate the payoffs of all the rounds they play, and we normalize this quantity by the total number of played games, such that each node's performance is represented by its average payoff.

The second part of each micro-step of the evolutionary dynamics of the game involves updating the strategy of node n_i. To this end, we normalize the discrete replicator dynamics for the case of higher-order interactions. We propose to compare the payoff π_i of a node n_i with the maximal payoff of the selected hyperlink l_i. Under this rule, n_i will adopt the strategy of the node with the maximal payoff of probability $\frac{1}{\Delta}[(\max_{l_i}\pi_j) - \pi_i]$, where Δ, whose precise definition is provided in Supplementary Equation 3, accounts for the maximal payoff difference, and is employed to guarantee that the probability is normalized. The rationale behind the choice of this expression is that node i will compare its payoff to that of the node with the largest payoff in hyperlink l_i. Note that the previous expression reduces to the standard one of the GI when $g=2$. Summing up, the HI accounts for a more realistic update than that in the BI, since the player inspiring a strategy change is the one with the highest payoff of the group, and not a randomly chosen one.

Uniform hypergraphs. To get some insights into the dynamics of the system in a simple configuration, we first studied the PGG on uniform random hypergraphs (URH) with hyperlinks of order equal to $g=2$, 3, 4 and 5 (see Methods for details on how to generate URH). Numerical simulations have been carried out for hypergraphs with $N=1{,}000$ nodes (players), and the game has been iterated for $T=10^4$ time steps. Figure 2a shows the final fraction of cooperators as a function of the reduced synergy factor r. In each case, the simulations refer to hypergraphs with $L=L_c$ hyperlinks, where L_c accounts for the minimal number of hyperlinks that guarantees the connectedness of the hypergraph. As it can be seen in the figure, there is a value of r beyond which cooperation emerges. We define this critical value of the reduced synergy factor, r_c, which depends on g, as the lowest value of r for which the fraction of cooperators is non-zero.

The results show that r_c decreases when the order g of the hyperlinks of the hypergraph increases. This is equivalent to saying that r_c decreases when the same number of $N=1{,}000$ individuals play in larger groups. We believe that this observation is important, since determining how r varies with the size of the group, allows us to get more realistic insights. Admittedly, the well-mixed limit of population-size groups is rarely applicable in reality, thus, the study of the impact of having large groups inside large populations, as allowed by our higher-order framework, is key. Figure 2b displays how the value of r_c depends on the number of hyperlinks L in the hypergraphs. For each value of g, we observe an increase of r_c with L, and a tendency, for large hypergraph densities, to the value $r_c=1$, which corresponds to the well-mixed replicator approximation[52]. The replicator equation approximation relies on the indistinguishability of the nodes, and as such, it is exact when the hypergraph is fully connected, that is contains all the possible hyperlinks. However, we show that the approximation is also good for sparse hypergraphs, with a number of hyperlinks of the order of the critical value for ensuring a giant component. Therefore it is natural that the higher the value of L, the closer r_c is to 1. The same argument can be used to explain the results in Fig. 2a. The ratio L_c/C_g^N, which represents the fraction between the critical number of hyperlinks L_c and the total possible number of hyperlinks, given by the binomial coefficient C_g^N, decreases with g. This implies that, if two hypergraphs have $L=L_c$, but different values of g, the one with lower g will be denser, and thus will exhibit a critical point closer to the analytic prediction. Therefore, we can say that at fixed reduced synergy factor, r, large groups are better to foster cooperation in sparse hypergraphs, as the number of hyperlinks required for connecting all the players represents a smaller fraction of the total number of hyperlinks. Finally, the value of r also influences how long it takes for the system to converge to the stationary solution. This is illustrated in Fig. 2c, where we report the measured relaxation time T from an initial configuration with $x_D = x_C = 0.5$, in a hypergraph with $L=5L_c$. These

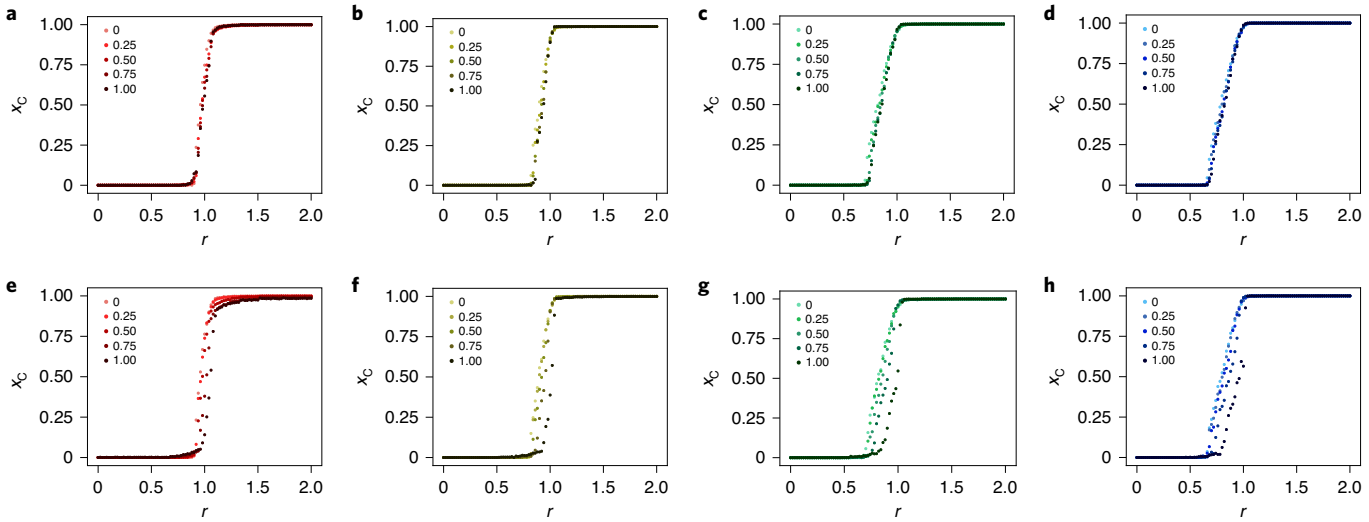

Fig. 3 | A PGG with higher-order interactions in hyperdegree-heterogeneous random hypergraphs. a–h, Numerical simulation for the fraction of cooperators x_C as a function of the reduced synergy factor r on hyperdegree-heterogeneous random hypergraphs of $N=1{,}000$ players and different orders g. Top and bottom panels refer respectively to PRHs and scale-free random hypergraphs (SRHs), while the four different columns correspond to values of g: **a**,**e** ($g=2$); **b**,**f** ($g=3$); **c**,**g** ($g=4$); **d**,**h** ($g=5$). Legends denote the value of μ characterizing the hyperdegree distribution, where larger values of μ imply higher heterogeneity.

results are obtained by running the simulations up to a maximum of 10^4 steps. Furthermore, for the replicator approximation, the value of T can be analytically computed as $T=\frac{\ln(N-1)}{|Q|}$, with $Q=(1-r)/\Delta$ (details of the calculations are shown in Supplementary Equations 4 and 5). As can be seen in Fig. 2, the agreement between the theoretical predictions and the numerical results is not only qualitatively but also quantitatively very good. The absorbing state, either full cooperation or full defection, emerges when the system is at equilibrium, a condition that can only be reached if enough iterations have occurred. On the other hand, real-world social interactions that can be modelled as games usually take place over a limited time interval τ. Hence, the relation between the relaxation time T, which depends on the synergy factor r, and τ is crucial to determine if the system does or does not reach the equilibrium, and consequently, if the replicator dynamics can or cannot predict the numerically computed fraction of cooperators. All these results indicate that the dynamics of the PGG on uniform random hypergraphs correspond to the replicator dynamics in the well-mixed limit. In order to test the robustness of these findings with respect to the implementation selection, we have also carried out numerical simulations in the BI implementation (Supplementary Fig. 1).

Hyperdegree-heterogeneous hypergraphs. The previous section addressed the simplest scenario in which the individuals of a population are assumed to be indistinguishable (URH). However, such an assumption can be oversimplified to describe real situations as it is well known that social systems are heterogeneous. Think of your friends at college. It is likely that a minority of them are involved in considerably more activities, and therefore social circles, than the rest. Such heterogeneity is typically characterized by a non-exponential degree distribution, allowing the presence of hubs, or highly connected individuals[45]. Hence, we consider here two families of hyperdegree-heterogeneous hypergraphs that we name power random hypergraphs (PRHs) and scale-free random hypergraphs (SRH). The algorithms we have used to generate these hypergraphs are reported in the Methods, and their properties have been studied in Supplementary Figs. 2 and 3. Scale-free hypergraphs are characterized by a power-law distribution, and represent the most hyperdegree-heterogeneous family of hypergraphs considered here. For this reason, these hypergraphs display a hierarchy between the nodes, as a few of them are involved in most of the hyperlinks and thus have a dominant position in the dynamics of the system. By contrast, PRHs stay in between uniform and scale-free hypergraphs, as their hyperdegree distribution combines exponential and non-exponential functions.

To study the emergence of cooperators in hyperdegree-heterogeneous hypergraphs, we have run $T=10^4$ time steps of the game on ensembles of hypergraphs with $N=1{,}000$ nodes and orders $g=2, 3, 4, 5$, respectively sampled from PRH and SRH. In order to compare the simulations with those reported in Fig. 2a, we have fixed the total number of hyperlinks to $L=L_c$. When, for high heterogeneity, some of the nodes (a minimal fraction of the total) do not belong to the main component, we have neglected their contribution to the fraction of cooperators.

The results reported in Fig. 3 show an important difference between PRH and SRH. In the case of PRH (top panels) the position of the transition does not depend on the heterogeneity of the node hyperdegree distribution, tuned by parameter μ, (see Methods for the precise definition of μ), and the critical point is the same as that obtained in URH. Conversely, the simulation of the game on SRH (bottom panels) shows that, the larger the heterogeneity in the hyperdegree distribution (larger values of μ), the more the solution deviates from that of URH, and the closer the critical point gets to $r=1$. This indicates that hierarchically structured systems inhibit cooperation in the PGG with higher-order interactions at variance with numerical simulations obtained on traditional networks under the same evolutionary dynamics.

In order to be able to explain these results we need to consider a refinement of the replicator approximation that takes into account the possible presence of correlations between the hyperdegrees of nodes belonging to the same hyperlink. Let $\mathcal{K}$ be the set of all possible hyperdegrees a node can have, and let $k\in\mathcal{K}$ be the hyperdegree of a randomly chosen node. We now denote as $p(\mathbf{k}''|k)$ the conditional probability that the node of hyperdegree k is part of a hyperlink where the remaining $g-1$ nodes have hyperdegrees $\mathbf{k}''=\{k_1, k_2, \ldots, k_{g-1}\}$, where $\mathbf{k}''\in\mathcal{K}^{g-1}$ is a vector whose $g-1$ components are elements of $\mathcal{K}$. We have been able to show that the system will fulfil the replicator approximation as long as the conditional probability $p(\mathbf{k}''|k)$ does not depend on k (see Supplementary Equation 6 and

the section below it for detailed analysis). This is true for the case of the PRH. Conversely, in the case of the SRH, increasing heterogeneity while maintaining the total number of hyperlinks in the hypergraph requires reducing the number of effective nodes. This induces non-trivial correlations in the model between the hyperdegrees of nodes belonging to the same hyperlink, and has a similar effect of driving the system closer to the $r_c = 1$ threshold, as that we have observed when we increase the hyperlink density in the uniform case (URH). Intuitively this can be explained by the notion of locality. When the density is low, or when no large hubs are present in the system, there is a non-negligible probability that cooperator bubbles emerge below the critical threshold, because there may be regions of the hypergraph that are semi-isolated, and therefore protected from defectors, even if they belong to the same component. However, either increasing the density or introducing hubs will reduce the probability of finding these isolated groups of nodes, and therefore will inhibit the formation of cooperator bubbles below $r = 1$.

Order-heterogeneous hypergraphs. Heterogeneity can also arise in the order of the hyperlinks. Indeed, the proposed HI of the PGG allows studying the more general, realistic and interesting case of hypergraphs where not all the hyperlinks have the same order. Important examples of such systems include teams of different sizes working for a common goal or one-to-many communication via apps like WhatsApp, where users can create and belong to several groups of different sizes. In what follows, we consider order-heterogeneous random hypergraphs with an assigned distribution of hyperlinks. Such hypergraphs are characterized by their total number of hyperlinks L and by a probability vector $\mathbf{p} = \{p^g\}_{g=g_-}^{g_+}$, whose entry $p^g = k^g/k$ specifies how likely it is, on average, that the hyperdegree k of the node contains k^g groups of order g. $\mathbf{p}$ is normalized such that $\sum_{g=g^-}^{g^+} p^g = 1$. Considering groups of different orders in the same hypergraph allows us to focus on another important aspect of the PGG on higher-order structures, namely, the possible dependence of the rescaled synergy factor r on the order of the group. This is important for practical purposes, given the increasing interest in understanding how the size of a group impacts its performance. As it has been shown recently[53], large and small teams play different roles in science and technology ecosystems. Thus, it is natural to assume that the synergy factor of a group depends on its size. This is particularly true in scientific publications, where it has been shown that the larger the group, the more citations a produced publication is likely to attract[54,55]. Therefore, as a general form for such a dependence we assume that the synergy factor R is an increasing power-law function of g, namely:

$$R(g) = \alpha g^{\beta} \tag{1}$$

with parameter $\alpha > 0$ and exponent $\beta \geq 0$. The value of the exponent allows tuning of the benefit so that the players are able to produce when working as a group. In particular, adopting a superlinear scaling $\beta > 1$, means considering a synergistic effect of a group that goes beyond the sum of the individual contributions[56,57]. Notice, however, that the assumed dependence in equation (1) is only a first approximation as it neglects saturation effects or even possible disadvantages due to difficulties in coordinating large groups, which, as we will see later on, appear in real systems. Under this assumption, the average payoff difference between cooperation and defection can be written as:

$$\pi_D - \pi_C = \sum_{g=g_-}^{g_+} p^g (1 - \alpha g^{\beta-1}) \tag{2}$$

where g_- and g_+ are again the minimal and maximal orders of hyperlinks, respectively. The relaxation time is again given by $T = \ln(N-1)/|Q|$, where $Q = (\pi_D - \pi_C)/\Delta$ (see Supplementary Equation 5 for the definition of Δ in the general case and for explicit calculations). It is then possible to derive the critical value of the parameter α as a function of the exponent β as:

$$\alpha_c(\beta) = \frac{1}{\sum_{g=g_-}^{g_+} p^g g^{\beta-1}} = \frac{1}{\mathcal{K}_\beta} \tag{3}$$

where, for simplicity, we have defined $\mathcal{K}_\beta \equiv \sum_{g=g_-}^{g_+} p^g g^{\beta-1}$. We remark here that $\alpha = \alpha_c$ for a fixed value of β is the critical point separating the defection and cooperation phases. This means that when $\alpha < \alpha_c$ the system will converge to full defection, while for $\alpha > \alpha_c$ it will converge to full cooperation.

To explore how the dynamics evolves in order-heterogeneous random hypergraphs, we have performed numerical simulations of the PGG considering four different values of $g = 2$, 3, 4 and 5 and allowing the values of p^g to take only multiples of 0.25. This leads to 35 possible hypergraphs, one for each of all conceivable convex sums of $\{p^2, p^3, p^4, p^5\}$ with the previous constraints. This means that the hypergraphs we consider are composed by hyperlinks of different orders, where each order g takes Lp^g hyperlinks out of the total number L. For instance, on a hypergraph with $L = 100$ and order probabilities (0, 0.25, 0.25, 0.5), on average we would expect 25 hyperlinks of order $g = 3$, another 25 of order $g = 4$ and the remaining 50 of order $g = 5$. Results are reported in Fig. 4 for four different values of the power exponent β, namely, $\beta = 0$, 1, 2, 3, shown with different colours. Notice that the case $\beta = 1$ corresponds to the underlying linear assumption of the standard PGG: in this case, α plays the role of the reduced synergy factor r. Figure 4a–d plot the colour-coded fraction of cooperators as a function of the parameter α in the definition of the synergy factor. The hypergraphs $\mathcal{H}_i$ have $\langle k \rangle = 2k_c$ and are displayed according to their value of $\mathcal{K}_\beta$, that is, the value of the critical point $\alpha_c(\beta)$. As for the case of uniform random hypergraphs, we find that although the critical point is slightly overestimated for low densities by the analytical approximation, there is still a good agreement between the theoretical predictions of the well-mixed replicator approximation and the numerical simulations. We next explore the behaviour of the relaxation time. Figure 4e–h shows results obtained for order-heterogeneous hypergraphs with $\langle k \rangle = 5k_c$. As was done for the homogeneous scenario, we follow the dynamics of the system up to a maximum of $T = 10^4$ time steps. The plots show that the relaxation times depend on α for all values of $\beta \neq 1$, albeit rather differently with respect to the dependence of the critical value α_c for $\beta < 1$ and $\beta > 1$. In order to further explore this relationship, we analysed how the average relaxation time varies as a function of the critical point α_c. Results shown in Fig. 4i–l reveal that the dependence is always linear. However, when the synergy factor increases super linearly, there appear different curves, each corresponding to a distinct family of hypergraphs and characterized by a different linear relation between the average relaxation time and the critical value. This behaviour introduces an additional degree of freedom that can turn very useful, since the degeneracy that is observed for $\beta \leq 1$ is broken for $\beta > 1$, and therefore one can independently set a critical point and a relaxation time by opportunely choosing the corresponding hypergraph. We remind the reader that cases with $\beta > 1$ are those in which the synergy factor $r(g)$ has a superlinear dependence on the order g. Those values of β are a priori the most interesting ones to study, and the ones more likely to be found in real situations. Therefore, our results about the relaxation are particularly relevant, because in this case one can potentially turn an unstable system into a stable one, and vice versa, by changing the order of the hyperlinks, while still respecting the value of the critical point.

Synergy factor of real games. From the previous results, a natural question arises: is it possible to determine the value of the synergy

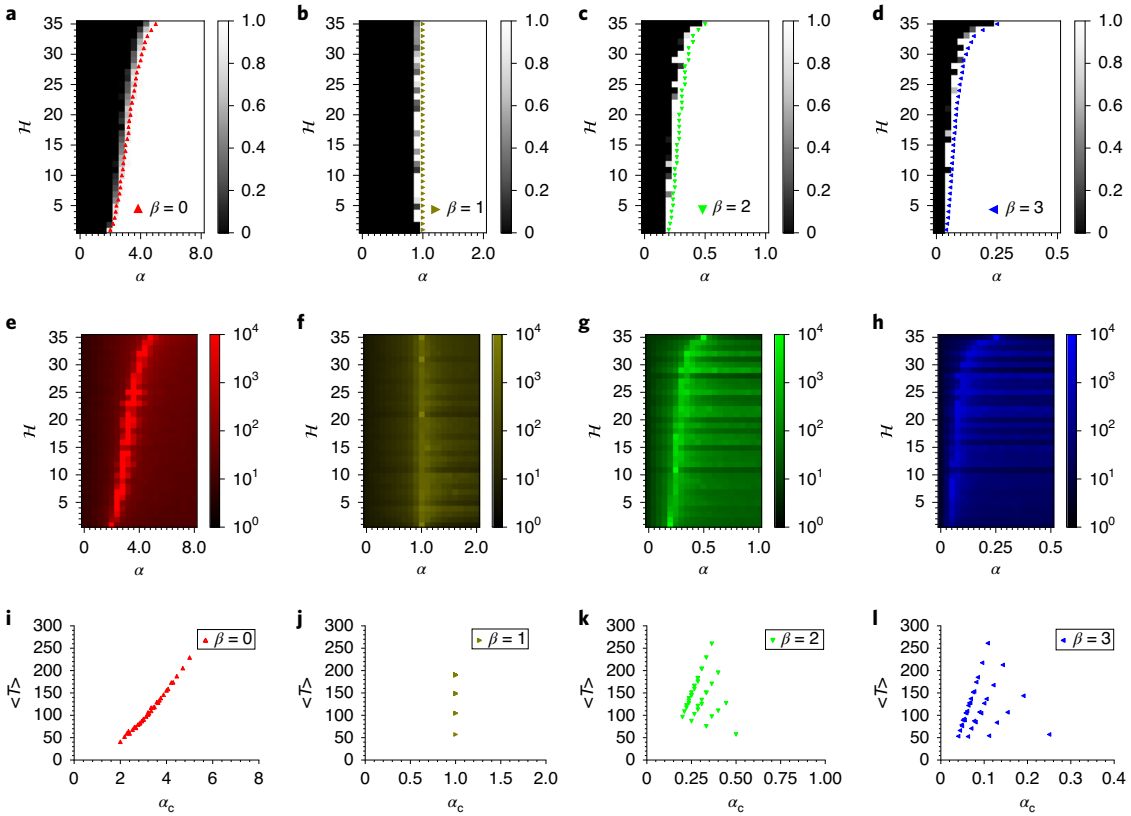

Fig. 4 | A PGG with higher-order interactions in order-heterogeneous random hypergraphs. We assume that the synergy factor grows according to equation (1) and consider the set of hypergraphs $\mathcal{H}_i$ that contain hyperlinks of orders $g = \{2, 3, 4, 5\}$ with probabilities p_g taking values in the set {0, 0.25, 0.5, 0.75, 1}—there are 35 possible such hypergraphs. **a**–**d**, Fraction of cooperators as a function of α for each of the 35 hypergraphs $\mathcal{H}_i$ and several values of β. The hypergraphs are ordered according to their value of $\mathcal{K}_\beta$. Simulations have been carried out up to $T = 10^4$ time steps for hypergraphs with $\langle k \rangle = 2k_c$, and triangles correspond to the theoretical predictions in the replicator approximation (Supplementary Information). **e**–**h**, Relaxation time as a function of α for the set of hypergraphs $\mathcal{H}_i$. Now hypergraphs have $\langle k \rangle = 5k_c$. **i**–**l**, Predictions for the critical value α_c as a function of the average relaxation time, calculated for each hypergraph in $\mathcal{H}_i$ by averaging over the intervals of $\alpha = [0,8]$, $[0,2]$, $[0,1]$ and $[0,0.5]$ for $\beta = 0$, 1, 2, 3, respectively.

factor for a real PGG for each of the possible group sizes? A plausible answer to this question can be obtained under the assumption that the very same structure of the hypergraph is the result of an evolutionary process in which nodes select the groups they belong to. We hypothesize that each individual tries to optimize the ideal number of groups of each order, based on the perceived dependence of the synergy factor on the group size. In this way, each real-world hypergraph would be the optimal structure that supports the game it hosts. We could then extract the functional form $R(g)$ directly from the hyperdegree distribution of the hypergraph. More precisely, the goal would be to use the information in the vector $\mathbf{p}$ of the hypergraphs on which the PGG occurs to determine the functional form, $R(g)$, of the synergy factor by imposing two conditions. The first condition comes from the assumption that the unknown reduced synergy factor $r(g)$ is proportional to p_g. This originates in the intuition that the distribution of the hyperdegree of a generic player should be aligned with the potential benefit that each player expects to obtain for each higher-order interaction. The second condition imposes that the average payoff of cooperators is equal to the average payoff of defectors. This implies that the system is at equilibrium and guarantees the coexistence of cooperators and defectors. Thus, given that these two conditions are satisfied, it is possible to extract the curves of $r(g)$ and $R(g)$ from empirical data on higher-order interactions.

In order to show how the above-mentioned procedure works in practice in real cases, we have studied collaboration in science and technology. We believe that this could constitute an example in which the benefit of a group depends on its size and at the same time, all group members do not contribute the same to the collective task, which essentially leads to a PGG dynamics. Although there is not a single way of classifying in a binary manner (either cooperator or defector) the authors of a scientific publication, one can think of two type of players mimicking cooperators and defectors. A cooperator can be considered as anyone that has contributed at least a 'fair' amount of work. The reverse applies to defectors, which can be considered those that put less effort in producing a teamwork than the average or the 'fair' amount of work. Note that whatever the effort of the team members is, they all receive the same benefit, for instance, in terms of citations (the citation is to the paper, not to the individual). Thus, given that there are cooperators and defectors, what is the optimal collaboration (group) size? And that of the synergy factor?

In particular, we have considered a large dataset of all the scientific articles published in the last century in thirteen journals of the American Physical Society (APS). For each journal, we have constructed a hypergraph whose nodes and hyperlinks represent respectively scientists and co-authored publications (Supplementary Table 1 has further details). The reduced synergy factors have then been obtained from information on the number of authors in each publication (see Methods). From the plots of $r(g)$ versus g reported in Fig. 5a we notice the existence of a maximum value of r at intermediate group orders g. This indicates that there is an optimal

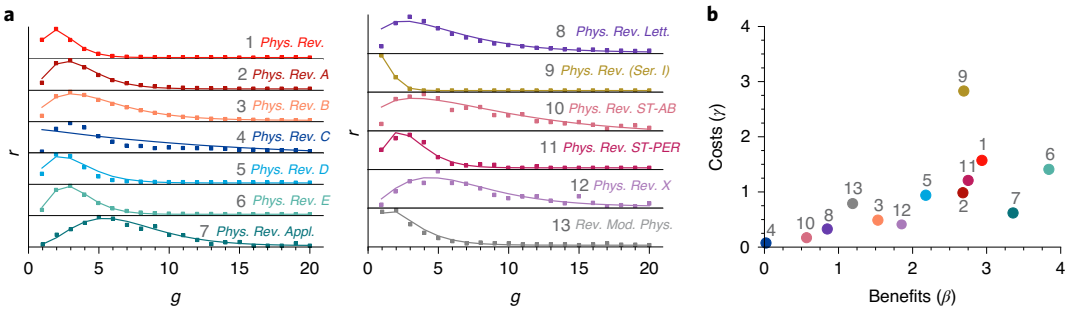

Fig. 5 | Synergy factors of scientific collaborations. a, Empirical synergy factors extracted from the structure of hypergraphs describing co-authorships from publications in the American Physical Society journals. Solid lines are fit of the empirical dots according to equation (6). Different symbols and colours refer to different journals, i.e., to different scientific communities. **b**, Journals' (labelled from 1 to 13 as per **a**) positions in the costs–benefit diagram. The synergy factors are factorized as a function of two competing terms, one modelling the benefits of cooperation, which is dominant for small group sizes, and another accounting for costs associated with an excessively large number of co-authors, which describes the exponential decays observed in the first two panels.

trade-off between the positive and negative effects of increasing the group size. The optimal value of g depends on the specific scientific community, as it varies from journal to journal. In the case of PhysRevLett the maximum of $r(g)$ is located at $g=3$. Different journals are associated with other optimal collaboration sizes. For instance, for *Phys. Rev. Appl.*, $r(g)$ is maximum at $g = 5$, indicating that larger collaborations are more beneficial in applied topics, such as device physics, electronics and industrial physics. For almost all journals, the synergy factor is low for $g=1$, showcasing the difficulty of publishing alone in physics, a research area where teamwork has been becoming increasingly important in the past decades[58]. Interestingly, a paradigmatic case is the one of *Phys. Rev. (Ser. I)*, the very first journal published by the APS in the early 1900s, for which a sharp peak is located at $g=1$, showing how most publications were produced by single scientists, in contrast with current trends. In order to shed light on this result we have factorized the synergy factor as the product of an increasing function of g times a decreasing function of g, and we have performed a numerical fit to extract the benefit exponent β and the so-called cost parameter γ (see Methods). This enables us to interpret the synergy factor as a combination of two contrary effects of the higher-order interactions in this particular dataset.

Figure 5b reports the values of β and γ obtained for each journal of the APS, and it allows us to classify the different scientific communities in terms of benefits and costs of higher-order interactions. These results provide a game-theoretic interpretation of the APS dataset. Specifically, in the context of this bibliographic dataset, hidden benefits and costs that conform to the synergy factor can be associated with several aspects of the task of producing a publication. Benefits (an increase of the synergy factor with increasing g) would correspond to the potential reinforcement of the amount and quality of the ideas and the potential increase in the outreach of the work with the number of co-authors involved. On the contrary, the costs (decrease with increasing g) would be the additional organizational effort in the process of arriving at a consensus and carrying out the tasks for publishing a paper. Experimental communities, such as that of nuclear physicists publishing in *Phys. Rev. C*, tend to have low costs. These ideas are aligned with recent studies about the creation and production of research ideas[59] and the role, group dynamics and success of teams[53–55]. Our formalism allows for a quantitative analysis of these phenomena and could be used in future applications to design ways to foster higher-order cooperation.

Discussion

Summing up, we have introduced higher-order interactions in evolutionary games to study cooperation in groups. Since higher-order interactions allow for a single link to connect more than just two individuals, they are naturally suitable to define groups in networks. In doing so, higher-order interactions thus do away with the arbitrary definitions of groups in classical networks, and they provide a formal theoretical foundation to study cooperation in networked groups. We have shown that the PGG on a hypergraph is effectively a null model that agrees exactly with the replicator dynamics in the well-mixed limit as long as no hyperdegree–hyperdegree correlations exist. As such, it can be used in future research towards upgrades that add additional layers of reality in models of human cooperation, either by means of strategic complexity[10], or by means of more complex interaction networks[60].

Towards the latter effect, we have also studied the PGG on hyperdegree-heterogeneous and on order-heterogeneous hypergraphs, where we study the effects of the presence of highly connected individuals and of hyperlinks of different orders, respectively. Due to the exact definition of a group in the proposed framework, we have been able to systematically and consistently consider synergy factors that are dependent on group size. Indeed, the framework allows us to unveil the effects of group size on cooperation in its most general form. As an example, we have considered synergy factors that are given by different powers of the group size, showing a critical scaling in the transition from defection to cooperation. In this case too, we have observed a substantial agreement between the simulations and the analytical predictions of the model. Interestingly, we found that hierarchically structured hypergraphs could hinder cooperation in a structured population. Our framework enables analysis of real systems, as we have shown for the APS publications dataset, providing insights regarding the positive and negative effects associated to higher-order interactions and the nature of group dynamics. However, even if our framework includes diverse forms of higher-order interactions, we recognize that a current limitation of this representation of human interactions is given by the constraints imposed by the available data. Admittedly, the identification of interactions in social networks beyond the traditional pairwise relationships constitutes an important challenge nowadays. Interestingly enough, this also represents an opportunity from an experimental point of view. It is also worth mentioning that the application of our results to scientific publications is based on the hypothesis that the interaction structure is the outcome of an optimization process, where the average distribution of groups that each node is part of coincides with the synergy factor, such that the system is in a stationary state of the dynamics of the PGG. This hypothesis, which constitutes a limitation of our method to extract synergy factors from real data, could potentially be either validated or refuted by models considering the dynamics within the topology

of interactions on top of the PGG. Moreover, the PGG imposes all cooperators to contribute with the same amount, making this contribution a boolean variable in practice. And therefore, an additional limitation arises when adapting real systems to the rigid formalization of the PGG, as the role of cooperators and defectors cannot be unequivocally defined when the contributions are not only 0 or 1.

It is also worth mentioning that in his essay titled *Innate Social Aptitudes of Man*, W. D. Hamilton wrote, 'There may be reasons to be glad that human life is a many-person game and not just a disjointed collection of two-person games.' He was referring to the fact that social enforcement works better in groups with more than two members, which can offer at least a partial cure for the problems with reciprocation in larger groups[38]. We note that the theoretical framework of higher-order interactions also invites to re-examine other fundamental mechanisms that may promote cooperation, such as image scoring[39–41], rewarding[61] and punishment[62–65].

Given the fundamental differences between pairwise and higher-order interactions, it would also be of interest to revisit the role of specific network properties and their role in the evolution of cooperation. In this regard, the role of community structure[21], as well as two or more network layers[22–29], promise to be fruitful ground for future explorations on how interaction structure impacts cooperation. Overall, we believe that the introduction of higher-order interactions to evolutionary games has the potential to improve our understanding of the evolution of cooperation and other social processes in networks.

Methods

Uniform random hypergraphs. We detail here the procedure we have adopted to sample g-uniform hypergraphs, that is hypergraphs with all hyperlinks of the same order g. A URH of order g can be constructed by assigning a uniform probability p to each g-tuple of $\mathcal{N}$. For each of them, a random number in the [0, 1) interval is generated, and if this number is lower than p, the hyperlink containing the g-tuple is created. However, this method scales badly with g since the number of g-tuples to be considered is equal to the binomial coefficient $C_g^N = \binom{N}{g}$, which grows fast as a function of g. A more efficient procedure is to fix the total number of hyperlinks, L, and generate a random integer in the $[1, C_g^N]$ interval. One has to provide an ordering for the set of all possible hyperlinks, so that each of the random integers corresponds to a hyperlink. The hyperlinks selected through this process are then added to the hypergraph. The hyperlink ordering is based on the following combinatorial identity

$$C_g^N = \sum_{i=1}^{N-(g-1)} C_{g-1}^{N-i}$$

that allows us to partition set $\mathcal{L}$ of all the possible hyperlinks of a g-uniform hypergraph in terms of disjoint hypergraphs, each one of them containing the hyperlinks that form the corresponding g-star hypergraph[66]. This holds true in general, which enables us to apply the same argument recursively, such that we can order all the possible hyperlinks univocally, and even more, the probability for having a specific node in a hyperlink is equal for all the nodes. These properties arise from the combinatorial probabilities $d_i = \frac{C_{g-1}^{N-i}}{C_g^N}$ for $i = 1, \ldots N-(g-1)$ that is the normalized weights of each of the terms in the summation. We have empirically found a distribution that can be used as an approximation to $c_i = \sum_{j=1}^{i} d_j$, the cumulative distribution of d_j, namely given by $1-(1-x)^g$, where $x = i/(N-(g-1))$. Supplementary Fig. 2 contains a numerical proof of the convergence between both expressions.

For the purpose of studying the stationary condition of a game, we are interested in having a connected hypergraph. The critical thresholds for the number of hyperlinks, L_c, and the hyperdegree, k_c, are equal to $L_c = \frac{N}{g} \ln N$ and $k_c = \ln N$. Hence, when L is larger than L_c, there is a high probability that the resultant hypergraph is connected.

Power random hypergraphs. We have seen that using the combinatorial probabilities d_i allows us to create uniform random hypergraphs. Therefore, increasing the value of the exponent g to g' in c_i, such that $g' > g$, will increase the probability of sampling the hyperlinks belonging to the g-star hypergraphs of low index nodes, and therefore introduce heterogeneities in the degree distribution. The control parameter that we use in the simulations in the manuscript is $\mu \in [0, 1]$. In terms of μ, one can obtain the power to use in the cumulative distribution g' as $g' = (1+\mu)g$. In order to sample hyperlinks of order g according to the new distribution, we transform the random number r to a different random r'

$$r' = c_{i-1}(0) + [r - c_{i-1}(\mu)] \frac{d_i(0)}{d_i(\mu)} \quad (4)$$

Here i is the g-star to which the hyperlink would belong if it was sampled according to r. In this expression $d_i(\mu)$ and $c_i(\mu)$ account for the distributions using the value of g' as a function of μ. Accordingly, $d_i(0)$ and $c_i(0)$ are simply the distributions of the uniform case. Supplementary Fig. 3 has an analysis of degree distribution emerging from the PRH.

Scale-free random hypergraphs. The standard indicator of heterogeneity in graphs is the power-law decay of the degree distribution. Here we employ the static scale-free algorithm[67] to generate such a profile. We use the same control parameter as in the PRH, $\mu \in [0, 1]$, which in this case results in a power-law $p_k \sim k^{-\lambda}$ where the power λ is $\lambda = 1 + 1/\mu$. Supplementary Fig. 3 illustrates degree distribution of the hypergraphs generated with the SRH.

Extracting synergy factors from real data. We show here how the dependence of the reduced synergy factor $r(g)$ on group size g can be derived for real systems, based on the assumption that this information is encoded in the very same structure of a hypergraph. In particular, we have considered a dataset of scientific publications and we have used it to investigate how benefits change with the size of groups in scientific collaborations. The dataset consists of 577,886 papers published in the period from 1904 to 2015 in the collection of all the journals of the APS[68]. We have constructed 13 hypergraphs corresponding to different journals, such as *Phys. Rev.* and *Phys. Rev. Lett.* of the APS. The nodes and hyperlinks of these hypergraphs represent scientists and publications respectively. The order of a hyperlink is equal to the number of authors of the corresponding publication. For each hypergraph, we have extracted the number L^g of hyperlinks of a given order g, which we used to compute the average number $k^g = gL^g/N$ of hyperlinks of order g that a node is involved in. The reduced synergy factor $r(g)$ can then be extracted from the proportion $p^g = k^g/k$ of hyperlinks of order g of a node, by assuming that $r(g) = zp^g$ and using the critical point relation:

$$\sum_{g=g^-}^{g=g^+} p^g (1 - r(g)) = 0 \quad (5)$$

to calculate the proportionality constant z.

Cost–benefit factorization of the synergy factor. In scientific collaborations across all journals of the APS, an optimal team size is associated with a maximum in the synergy factor, suggesting that an excessively large number of co-authors might lead to disadvantages in cooperation. In order to account for these effects, we have modelled the synergy factor extracted from real-world collaboration data as the following function of g:

$$f(g, \alpha, \beta, \gamma) = \alpha g^{\beta} e^{-\gamma(g-1)} \quad (6)$$

ruled by the three parameters, α, β and γ. The first parameter, α, introduced in equation (1), is determined by the critical point condition. The remaining two parameters account, respectively, for the benefits and costs of the higher-order interactions. Benefits are modelled as a power-law of the group size g with an exponent β. Costs are described by an exponential decrease in the group size tuned by the cost parameter γ. Different functions of g might also provide a satisfactory fit of the data. Here we have opted for this expression because it enables us to factorize the group size dependence into two different contributions, benefits and costs, which can be interpreted in terms of behaviours of the players. The benefits grow as g^{β}, where β captures the synergistic effect of group interactions. The term due to the cost associated to task organization in groups has its maximum at $g = 1$, and the exponential dependence has been adopted to avoid possible singularities of other functional forms at $g = 1$. In conclusion, equation (6) has a maximum at $g = \beta/\gamma$, which summarizes the result in a compressed expression. To extract the pair of parameters (β, γ) for each journal, we explored the parameter space and performed an optimization in order to reproduce the empirical points correctly. For each considered pair (β, γ), we computed the normalized distance between the synergy factor inferred analytically and the one associated to the data (Supplementary Equation 7 has further details on the procedure). The pairs with the smallest distance were selected as the outcome of the optimization process and are those reported in Fig. 5c.

Reporting Summary. Further information on research design is available in the Nature Research Reporting Summary linked to this article.

Data availability

The APS dataset is provided by the APS at: https://journals.aps.org/datasets.

Code availability

Custom code that supports the findings of this study is available from the corresponding author upon request.

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Author contributions

U.A.-R., F.B. and V.L. conceived the study with contributions from G.F.A., M.P. and Y.M. U.A.-R. performed the calculations. U.A.-R., F.B., G.F.A., M.P., Y.M. and V.L. analysed the data and discussed the results. U.A.-R., F.B., G.F.A., M.P., Y.M. and V.L. wrote the paper.

Competing interests

The authors declare no competing interests.

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